Game Theory and the Problem of Decision–Making

Andrej Démuth

Edition Cognitive Studies

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Content

Introduction

The following text aims to offer an introduction, in terms of basic ideas and terminology, into game theory and decision processes. Although the subject being presented here is mostly a domain of mathematical, economical, statistical, political or psychological approaches, this approach is attempting a philosophical analysis of applying rationality in decision processes. Therefore, contrary to other introductions into game theory, we will find no formulas, equations or mathematical symbols describing calculations of payment functions or probabilities of occurrence of certain phenomena in this work. Consequently, in Binmore style, I would like to apologize in advance to all who were hoping to find here exact, mathematically identifiable formulations of the effectiveness of certain strategies and profitability of given techniques. However, the absence of „mathematisation", as I hope, does not necessarily cause a decrease in accuracy and applicability of presented thoughts. On the contrary.

This text is trying to open up a whole area of thinking which includes a wide palette of problems and approaches hidden under one common name „The Game Theory" to those interested in the humanities. The reason for this step lies not only in the fact that the author himself is oriented toward the humanities (Philosophy/Psychology), but mainly because he strongly believes in the benefits and meaningfulness of showing the whole and the structure rather than the details (Kolmogorov Complexity). After all, the main target of any introduction into a subject is to present the gist of a problem, it's main elements and relationships within. Those interested in a deeper analysis of this issue can clearly find more detailed, mathematical models of explanation with more nuance.

Another reason for this humanistic approach is a personal experience with using game theory and decision processes in non–mathematical everyday situations. Undeniably, the motivation for this arose from the work of Nils–Eric Sahlin, a philosopher and teacher of game theory and the problem of doctor's risk at the Medical Faculty of Lund University (the chief of Medical Ethics). He showed me, that in medicine we encounter a number of problems and fatal decisions with a large degree of uncertainty or risk while part of the decisions can have (and often do) irreversible consequences. The same applies to other areas, such as the law, science, etc.

I think that the main advantage of game theory lies in its applicability and portability of its principal knowledge into various rare as well as common decision processes. For this reason the problem of deciding and analyzing the optimal strategies is an important part of any cognitive oriented studies. While learning certain knowledge, we shouldn't concentrate only on learning the main elements and the mechanics of this knowledge, but mostly on drawing the consequences from the knowledge we gained. This applies even more in cognitive studies that are part of the humanities.

As a consequence, the goal of this text is not only to explain the basic theories, terms and elements of decision processes, but also to encourage readers to search for application of presented processes in a wide spectrum of everyday situations — beginning with social and economic behavior through biology, ecology, ethics and finally in optimization of intelligent devices and systems. The structure of the text follows this goal. The chapters explain mainly the basic terms (written in italics as key words), theoretical frames and problems, but also the issues of possible application and include further examples which are part of recommended texts at the end of each chapter. However, the recommended texts also

point to some classic works or chapters in commonly used, mainly English–language university textbooks. I can not omit mention of Binmore's trilogy about rationality in a social context, which I strongly recommend for further study of evolutionary aspects of social behavior, as well as Petersen's Introduction to Decision Theory, which together with Dixit's Games of Strategies present the core of mapping the relevant topics of the presented text. The final part of this work includes a list of the most commonly used games in game theory and the list of relevant freely available sources for further study.

In Trnava, July 2012 — Andrej Démuth

1. Game theory and the problem of decision-making

Key words: *game theory, the subject, the purpose, areas of research, the history*

1.1 What is Game Theory?

Game Theory (GT) is a scientific field dealing with the study and analysis of strategic, rational decision processes of individuals and their interactions in a (social) environment. It concentrates on the study of "*ways, in which strategic interactions among economic agents produce outcomes with respect to the preferences (or utility) of the agents*," (Ross 2011) regardless of whether the results of their efforts were intended. Thus, we are mostly talking about "*a study of mathematical models of conflicts and collaborations between intelligent, rationally deciding subjects*" (Myerson, 1001, 1). "*Game theory provides general mathematical methods for analysis of situations, in which two or more individuals make decisions that influence the status of one or the other(s)*" (Meyerson, 1991, 1). The subject of its study is the understanding, explanation or prediction of results of a potential individual's interaction with the surroundings, in all the places where we suspect that the intention of one's actions can be modified by rational behavior of another individual or the environment, whose manifestations are also predictable.

1.2 Why study GT?

GT enables us to explain and predict behavior of individuals and systems, whose decisions and behavior assume the presence of rational algorithms for problem solving. GT therefore enables us to explain strategies and behavior of living and non–living systems (computers, IT, AI) leading to a fulfillment of a certain (conscious or non–conscious) goal. Considering the rationality and predictability of used algorithms for problem solving, it provides a possibility to predict the result of a planned activity depending on whether information entering the decision process (or participating in this process) is complete or incomplete. Because it is quite descriptive and also normative, GT can be applied in all areas, where it is possible to (relatively accurately) assess the probability and the weight (and perhaps the accuracy) of certain factors of the decision process as well as in areas, where the outcome of an action is not a result of pre–determined action, but it allows for a certain amount of choice. GT can therefore be used to explain relatively simple issues such as finding a favorable option involving a minimal number of variables (logic, mathematics, informatics) as well as explaining relatively complex problems like social behavior, economic behavior, ethics, biological theory of choice and many others.

1.3 Who studies GT?

GT is part of a wider group of theories of rational decision making (together with the theory of decision making and the theory of social choice), attempting to understand, explain and predict the result of an action. While decision theory accentuates mostly exogenous factors of a decision (status that is supposed to occur) GT concentrates more on (internal) intentions of other participants of a process. These intentions can be thought of as internal non–visible factors. Therefore we can say that GT is a theory focusing on situations where the outcomes of given interactions are consciously or non–consciously “negotiated” by those participating in them. In GT, the point for the participants is to predict the result of one’s own actions depending on the thinking and deciding of other agents through assessing the probability of reaching a certain goal through anticipation of decisions made by other participants in a given decision process (Grune–Yanoff, 2008). In other words, the point is to explain or predict the result of a given strategy of decision making and acting in a situation, when not knowing how exactly other participants would act, but assuming they are also using rational strategies of behavior.

This definition of GT results in a fact that in the forefront of its interest are rationality, intelligence, decision making, strategic behavior and assessment of risks and benefits. We can therefore assume that it will most likely become a subject of study for mathematicians, psychologists, sociologists and economists or all those who examine the determinants of human behavior. In this sense GT is an inter– and multi–disciplinary subject.

1.4 History of GT

The beginning of GT as a separate subject of study can be dated back to the first half of the 20th century and it is connected with the development of mathematical aggregates theory and mathematical economy (Dimand, Dimand, 1996). John von Neumann, one of the first six professors of mathematics at IAS Princeton (together with A. Einstein) is considered to be the founder of GT (Hykšová, 2012, 6). In 1928 he published the work *Zur Theorie der Gesellshaftsspiele* (Neumann, 1928, 295 –320), in which he used the theorem of Brouwer’s fixed point for continual mapping of compact convex groups. Subsequently in 1994 he and Oskar Morgenstern published the book *Theory of Games and Economic Behavior*, in which they tried to solve cases of cooperating strategies of behavior and in its second issue they laid down the axiomatic foundations and methods of GT, which were supposed to find its

application mainly in the area of economy. Despite questionable success of the original model of GT in the economy of the sixties and seventies, GT provided a general framework for explanation of economical behavior (in 1994, the Nobel Prize in economics was given to John Nash, Reinhart Selten and John Harsanyi, in 2007 the Nobel Prize in economics was awarded to Leonid Hurwicz, Eric S. Maskin, Roger B. Myerson for their mechanism design theory, which is a branch of GT). In 1970 the general model of finding evolutionary stable strategy in biology was also rewarded. It’s author (John Maynard Smith) received the Crafoord Prize in biology for this concept.

Although GT is a relatively young scientific discipline, it’s historical roots reach far into the distant past and by its principles are closely related to philosophy. The first encounter with GT can be found in antiquity and is related to fighting and strategic behavior. Each war as well as it’s imitation in game or sport assumes an understanding of a theoretical framework of GT. A clear example of this is the game of chess. The essence of success in this more than 2600 year old game (Murray, 1913) is the ability to anticipate the behavior (moves) of the opponent in advance and to adjust one’s own behavior accordingly. Both players understand that their overall intentions (to win) are identical, but the algorithm of steps, which should lead them to it, must be quite different. To penetrate the opponent’s internal strategy (or to come up with one, which will not allow him to develop his own) is the key to success. The tradition and teaching of chess shows that the implicit foundations of GT were known to people even in antiquity.

The first philosophical — theoretical analysis of GT can be, however, found (according to Don Ross) in Plato’s dialogs *Laches* and *Symposium*. In Socrates’ reflections of the battle of Delia, Ross and others analyze options of soldier’s behavior in the upcoming battle and the need to limit their possibilities of choice (to prevent them from running away), which markedly limits the fighting capacity of the army. M. Peterson, however, finds the roots of GT in Herodotus and Aristotle (*Topics III* — Peterson, 2010, 11). Similarly, Cortez's burning of his own fleet before the eyes of the Aztecs makes it impossible for Cortez's soldiers not to fight (either the Spanish win or they must die as there are now no ships to retreat back to). This paralyzes the natives because the Spanish demonstrate their confidence stemming from total bet (Ross, 2011).

The art of mobilization of courage and intentions was brilliantly described by Machiavelli, whose The Prince (*Il principe*) is a great example of thorough anticipation of desired behavior of subjects based on the behavior of the ruler. Machiavelli, similar to Hobbes in later years, understood that it is the achievement of subtle equilibrium between the interests of individuals in a society and the legal mechanisms that is the foundation of the existence of state and law. The concept of a natural social contract assumes that individuals who are equal eventually come to the understanding of the demands of common sense and to rational calculation, which tells them that it is more beneficial to them to give up some of their natural rights (for example the right to kill or steal...) — assuming reciprocity — in favor of finding sustainable peace, which is preferable to the individual. The moment of mathematical calculation of well–being became the foundation of the whole tradition of a utilitarian understanding of ethics, which showed in its main slogan "the greatest happiness for the greatest number" and it remains today as the background of the idea of searching for establishment of a just state and law.

Another example of developing the explicit forms of GT is Smith's philosophical–economic theory. The physiocrats and the proponents of the Laissez–faire movement discovered the rationality of nature and the self–organizing nature of our economic behavior. The essence of Smith's work *An Inquiry into the Nature and Causes of the Wealth of Nations* is an image of an invisible hand of the market and synergistic effect of egoid and compassionately oriented strategies leading to the improvement of the economic environment. Smith's work can be, therefore, considered as the precursor of economic GT in its latter–day representation.

A formalized version of the basic problem of GT can be found in Pascal's work. Pascal's bet proves the limited competency of the brain to prove the existence of God, but an excellent competency of the brain to correct the consequences of belief or disbelief in God. If I believe in God and he doesn't exist, my belief limits some of the small pleasures I could otherwise enjoy. If I don't believe in him and he doesn't exist, I enjoy the pleasures. But if God really exists and I believe in him I gain the whole of eternity in the form of redemption. And if he is and I don't believe in him, I lose everything. Pascal in his aphorism showed that since we are not able to make a decision about God's existence, it is more beneficial for us to believe in him because the possible consequences of this attitude correspond with our intentions more than the consequences of the opposite approach.

Although Pascal together with Pierre de Fermat can be viewed as the predecessor of the Port–Royal school of logic, which was solving a problem of mathematical probability in ethics similar to Daniel Bernoulli (Petersen, 2010, 12 — 13), James Waldegrave, who in 1713 published a discussion about mixed minimal–maximal strategy in the card game *le Her*, is considered the first immediate predecessor of mathematical GT. The fourth president of the USA and "the father of the constitution" James Madison published at the beginning of the 19th century a deliberation about different ways of behavior in various tax systems (Rakove, 2007), which became the foundation of Antoine Agustin Cournot's work *Recherches sur les principes mathématiques de la théorie des richesses* (1838), explaining the analogy of Nash's equilibrium. In 1921 (seven years before von Neumann's first essay) a French mathematician Émil Borel published a series of essays *La théorie du jue*. He analyzed (as von Neumann did later on) the behavior of a poker player while bluffing. Unlike von Neumann, he didn'd develop his theory into the theorem of "minimax", which (after it was translated by L. J. Savage) decided von Neumann's primacy.

1.5 GT and philosophy

The significance of philosophy for GT lies not only in the formation of its historical foundations. David Hume (as well as Franck Ramsey later on) also addressed the issue of probability of reasoning and predicting. Ramsey's breakthrough work *Truth and probability* (1926) provoked a big debate among Cambridge analytical philosophers (Russell, Moore, Wittgenstein) that led to a formulation of the prisoner's dilemma (Peterson, 2010, 13).

> Philosophers were voicing their opinion of the process of risk evaluation and optimization of benefits even after Neumann's formulation of GT. (Russell's chicken dilemma). Besides finalizing and modifying certain problems of risk evaluation, one of the key tasks of philosophers is a precise terminological clarification of factors which are part of the decision process and accurate understanding of rationality and of the decision process itself.

1.6 Recommended literature

ROSS, D.: „Game Theory" in: *Standford Encyclopedy of Philosophy*. http://plato.stanford.edu/entries/game–theory/, Part 1: Philosophical and Historical Motivation

MYERSON, R.B.: *Game Theory: Analysis of Conflict*. Harvard University Press 1991, Chapt. 1: Decision — theoretic foundation, 1 — 9.

DIMAND, M., A., DIMAND, R. W.: *The History of Game Theory*, Volume I. From beginnings to 1945. New York: Routledge Research 1996, Introduction: Defining Game Theory and its History, 1 –17.

WALKER, P.: A *Chronology of Game Theory*. [online]: http://www.econ.canterbury.ac.nz/personal_pages/paul_walker/gt/hist.htm

GÄRDENFORS, P., SAHLIN, N.–E.: Introduction: Bayesian decision theory — foundations and problems. *Decision, Probability, and Utility: Selected Readings*. New York : Cambridge University Press, 1988, 1 — 18.

2. Terminology

Key words: *game, strategy, utility, status, risk*

2.1 Game

The title designation game refers to any conscious and targeted interaction of an individual with its environment. The goal of this interaction should be a saturation of the individual's intentions. According to Jespers Juul important conditions of a game are as follows: 1. it is a formal system based on rules, 2. it has variable and quantifiable outcomes, 3. different outcomes have different values, 4. the players influence the outcomes through their behavior, 5. the players are emotionally involved in the outcomes, 6. the consequences are optional and transferable (Juul, 2005, 6 — 7, paraphrased by Mäyrä, 2008, 34 — 35). It follows that a game is an interaction between (mainly) rational players, but in principle it is not tied solely to the behavior of an individual toward another individual (or his group). The game can also be a behavior of an individual toward a system of relationships or entities, which react to the behavior of this individual and they enter the realization or non–realization (they obstruct or optimize it) of his intentions. These games without a specific opponent in the shape of a rationally thinking subject are called games with one player or games with Nature (Binmore 2011, 25) — Nature becomes the player. Strictly speaking: Nature is always a player (it influences the existence or non–existence of decision–makers as well as the basic rules), but because of the large number of games we can eliminate Nature's effects and concentrate on the behavior of other decision–makers.

"*A game happens when several players must make a decision in a situation in which outcomes for each player are partly determined by choices made by other players*" (Binmore, 2011, 25). In other words: a game can be any conscious behavior of a subject, which, based on this choice, leads toward a realization of a certain goal while understanding that the achievement of this goal will be influenced by choices of other actors.

2.2 Choice and strategy

A targeted behavior of a participant is an important condition of a game. Some philosophers call this a strategy. *Strategy* is a targeted choice made by a player leading toward certain goal. We should distinguish between a choice and a strategy. This differentiation is influenced by the characteristic of a game. In non–iterated games (one–shot games), where players make an individual decision in a unique situation (which doesn't repeat) we talk about a choice or a decision. In games with repeated moves, where "*a player who goes later can react to what other players before him had done (or he had done himself) in previous moves*" (Dixit, 2009, 27), we talk about strategies not only because it is a longer term (or also pre–meditated) activity, but mostly because strategy (compared to tactics with shorter duration and a smaller number of variables) takes into consideration more variables in regards to different consequences of opponent's moves or one's own previous decisions.

2.3 Probability and the theory of possible worlds

The choice and the strategy confirm that with games we must talk about the realization of a possible decision process. The existence of freedom (or at least a chance) in a decision process and a possibility to fulfill or not fulfill a goal is an essential condition of GT. If our decision inevitably leads to the same result (we suffer from omnipotence and there is no obstacle to the achievement of the desired status or, the opposite, our decisions never lead to the fulfillment of our intentions) the decision process is in principle irrelevant and the achievement of the desired status is independent of the decision process. In such case we can't talk about a game. A game supposes an ambiguous cause–effect connection between a decision and a result and this connection can be expressed by probability (which amounts to neither zero nor one hundred percent) of achievement of the desired state depending on our decisions and the decisions or interactions of other participants. The uncertainty and the possibility of various results points to a philosophical problem of possible worlds. We can talk about a game, when there is a possibility for the existence of various final outcomes. While taking no account of the historical and metaphysical aspects of the theory of possible worlds (Leibniz), the consideration of possibility and efficiency of outcomes is the main feature of any game.

2.4 Outcomes

A game should result in the achievement of the greatest possible utility. *Utility* is a degree of achieved outcomes when using certain choice or strategy. Outcomes are situations, which occur as a result of using certain choices or strategies. If these situations are consistent with our intentions, we talk about winnings (payoffs) if they are in conflict with the desired status, we talk about losses. This is independent of how these situations are viewed by other decision–makers or those who assess them. This shows that outcomes, goals and profits are the main engines of games although their assessment can be quite relative. As a result we can use different criteria and scales for their measurement: *ordinal scale* — comparing qualitative levels of statuses (e.g. grades at school), *cardinal interval scale* — quantitatively comparing objects to each other (e.g. the temperature of an entity), *cardinal ratio scale* — comparing the ratio of quantity of the criteria which is being considered (e.g. the ratio of revenues). The accurate quantification of profits and losses fundamentally influences the results of rational decision making in GT.

According to the distribution of possible utility among participants we can divide games into two groups. Zero sum games and non–zero sum games. In *zero sum games* the sum of gains and losses of all participants equals zero. It means that one participant's gain inevitably leads to another participant's loss (win–lost games). The amount of profit is equal to the amount of loss. In *non–zero sum games* we can see an asymmetrical distribution of profits and losses (win–win game) or the total sum of profits and losses depends on specific strategies used by particular decision–makers.

In repeated games it is quite evident that the total sum of profits and losses depends upon the sum of individual outcomes. Thus, the player adjusts his strategy so as to prioritize total gains over partial profits in individual games (moves). *Expected payoff* in individual games can be calculated as the sum of probabilities of occurrence of certain alternatives of results and the amount of profits gained in these alternatives. If a decision leads to a profit of 10 Euro with the probability of 10 % or to a loss of 1 Euro with the probability of 90 %, expected gain can be calculated as (10x0.1+(–1x0.9)) which equals 0,10 €. If a different decision's calculated expected gain is more then 0,10, we should obviously prefer this decision.

2.5 Points of departure

One of the crucial moments in GT is an accurate assessment of the starting point (*point of departure*). We mean realities of the world

(situations) which are independent of the decision process and chosen decision or strategy and of the amount of possible gains. On the contrary, these situations actually directly determine them. Thus, points of departure can be described as tasks of the problem which is being solved, explicitly or implicitly expressed relations between elements and values of the decision process. The goal of GT is to describe relations between points of departure, strategies and outputs and to find optimal strategies of behavior which would lead to the highest possible gain in a situation where the result of decision processes of other decision–makers is an unknown variable.

2.6 Deciding with certainty, with imperfect information and with risk

Total knowledge of points of departure enables the decision–maker to find an optimal strategy of solution. Philosophers believe that each game has an optimal strategy of solution, which reflects all the possible variables of the system. The problem is that we don't usually see all the possible variables. But when each of all possible alternatives of behavior leads to a known, definite consequence, we talk about *deciding with certainty* (Hykšová, 2012, 15). If individual alternatives of possible behavior lead to consequences, whose occurrence can be expressed through mathematical probability, we talk about *deciding with risk*. A good example of this is a lottery, Black Jack or roulette. The occurrence of a certain final state can be calculated based on a certain distribution of probability. But when this probability is not known or its value is uncertain (or without clear meaning), we talk about *deciding with uncertainty*. An example of this type of decision–making was Columbus' choice for the journey west (he knew neither what awaited him there nor with what probability) or a simple choice of a meal in a restaurant when we have absolutely no prior experience with this type of meal (should we order an unknown specialty or go with the classic? — Peterson, 2010, 1, 40). Both types of decision processes (with risk, with uncertainty) require from the decision–maker a different strategy of minimizing unwanted outcomes.

2.7 Rational deciding

The basic condition for the success of GT is the presumption of rationality in decision–making. Rational decision–making is a decision process in which all reasons lie solely in rational (deductive, inferential and computing) operations. Irrational decision–making doesn't (exclusively) follow rational motives.

In common human life, the rational and irrational merges into a continuous flow. As a result we can name predominantly rational and predominantly irrational reasons for decisions. In GT any decision–making based on other than exclusively rational motives is defined as irrational. If a player is indifferent to the outcome or the achievement of it is impossible, we talk about an irrational player.

Since the times of Pythagoras we define rational numbers as numbers that can be written as a simple fraction of two integers. Numbers that can't be written as a simple fraction, are irrational (Binmore, 2011, 1). This ability to logically express something defines rationality. Rational decision is, therefore, defined as a decision, whose reasons and consequences can be logically and consistently explained to another reasonable being.

> *While accepting universal rationality we can suppose that if subjects are deciding purely rationally, in identical situations (same access to information, same goals) they arrive at identical decisions.* In this sense rationality is a sign of stability and coherence of decisions, (Gilboa, 2010, 5) and not a sign of accuracy of these decisions. "*Decisions can be rational without being correct and they can be correct without being rational*" (Peterson, 2010, 4). GT studies purely rational decisions of players.

2.8 Recommended literature

MÄYRÄ, F.: *An Introduction to Game Studies. Games in Culture*. London: SAGE 2008, 32 — 51.

PETERSON, M.: *An Introduction to Decision Theory*. Cambridge University Press, 2010, 1 –30.

DIXIT, A., K., NALEBUF, B., J.: *Thinking Strategically: The Competitive Edge in Business, Politics, and Everyday Life*. New York: W. W. Norton & Company 1993, 1 — 6.

COLMAN, A., M.: *Game Theory and its Applications, In the Social and Biological Sciences*. London and New York: Routledge 1999, 3 — 14.

3. Classification of gt and basic problems

Key words: *mono–criteria, taxonomy, chess, order of moves, Zermelo's axiom*

3.1 Classification of GT

Any taxonomy is dependent on the choice of criteria. If there is just one criterion, it prevailingly leads us to binary mono–criteria classification. With more criteria we come to a multi–criteria classification, which could again be broken down into mono–criteria classification.

In literature we come across several classifications of GT. The following is a modification and completion of J. Friebelova's and M. Peterson's classification (Friebelova, 2006, 1 — 3 and Peterson, 2010, 220 — 224):

When classifying games based upon the *number of players*, we can have *games with two players* — a game with one player against Nature is a game with two players (rational decision–maker vs. Nature) — or *games with more than two players*. The number of players can be finite or infinite.

When considering the *total sum of payoffs*, we can have *games with a constant sum* and *games with a varied sum*. A specific example of a constant sum game is a *zero sum game*.

The next distinction (based on the *number of possible strategies*) is between *finite* and *infinite games*. Finite games are represented by a limited number of moves (e.g. game of stone–paper–scissors).

In indefinite games the player is choosing from an unlimited number of moves (e.g. Think of any number!). Based on the *number of realized choices* we can have *non–iterated games* (without repeating) and *iterated games*. Iterated games can be played with *the same opponent* or the same game can be played against *more changing opponents* (tournament).

In iterated games it is also important whether the *order of moves* affects the outcome of the game. If not, we talk about *normal games*, in which players make choices *simultaneously* or without knowing what the other player will do. If the order of moves is important and it influences an opponent's next move, we talk about *extensive forms of games (sequential)*.

According to a *player's interests* we can have *antagonistic games* and *non–antagonistic games*. In antagonistic games the main idea is to win at the expense of the opponent. In non–antagonistic games any positive payoff of the opponent is irrelevant for the choice of player's strategy, although even here we can divide games into *cooperative* and *non–cooperative*. In cooperative games the payoff of both players maximizes in a situation of cooperation, in non–cooperative games the payoff is independent of the cooperation.

Depending on *used strategies* we can have *symmetric* and *asymmetric games*. If all players are choosing the same strategy and gain the same payoffs, we talk about symmetric games. The specific individuality of the player, the order of the move, etc. are not important. In asymmetric games individual players can't use identical choices, or the same choices don't lead to identical payoffs.

According to the *consequences of choice* we can distinguish between *deterministic games* and *stochastic games*. If our choice directly influences the amount of payoff we talk about a deterministic game. In stochastic games a chance affects the result and the amount of payoff has a probability distribution.

Based on the *extent of information* we can talk about *games with perfect information* vs. *games with imperfect information*. The games with perfect information can be defined as games in which a player has the complete information necessary to adopt the right strategy (for example in the game of chess). No information is unattainable. In games with imperfect information a player lacks the information about some important variables (for example a game of ships). An example of an imperfect information game is deciding with risk and deciding with uncertainty.

A specific problem is the *method of information distribution*. For the most part we suppose that all players are equally informed. This status is called *games with equality of information*. However, there are also *games with asymmetric distribution of information*, which give some participants an advantage over the others. An example of this unequal access to information is guessing, interrogation or bluffing. Another example is signalization of some information by one member of a cooperating coalition within (or outside) the rules of cooperation.

Based on the *form of rules* we can divide games into *games with explicit and games with implicit rules*. Logically, games with explicit rules contain a full list of clearly and unambiguously formulated rules while games with implicit rules pre–suppose the existence of commonly accepted rules or a mechanism for negotiating them.

When considering the *stability of rules* we can differentiate between *games with constant (predominantly explicit) rules* and *games with an option for rule modification* (evolutionary and dynamic games).

If a player (in regard to the stability of a strategy) uses in each game (or in one round of a game) the same strategy for achievement of optimal payoff, we talk about *pure strategy*. If the player is forced to change his strategies, we talk about *mixed strategy*.

Based on the *type of recording*, we can distinguish between games which are recorded in the form of a *table*, games with *matrix* recording and recording in the form of *tree branching*.

As most games satisfy more different criteria in many games we can encounter several aspects and attributes of classification. In this sense one of the best researched games is that of chess.

Chess belongs to the category of two player games with constant zero sum. A game can end with a win of one player while the other loses, or with a tie (stalemate, threefold repetition draw or an agreed draw). It is an antagonistic game with a deterministic understanding of the influence decisions have upon their consequences. The rules of this game are clearly and explicitly set without the possibility for change. It is assumed that both players are equally aware of the rules and the situation on the playing board does not handicap (in the sense of having the same access to information) any of the players. Information, therefore, is distributed symmetrically and with chess we talk about a game with perfect information. But this applies only to the assignment of the game or the situation on the playing board.

Players have at their disposal a finite number of possible moves, which is determined by a finite number of playing figures (16) on each side (6 values) and by a finite number of squares on the playing board (64). At the beginning of a game, a player can choose from 20 possible moves and his opponent can react with one of 20 other moves. After the first move there can be one of 400 possible positions on the playing board. In the next move the number of possible positions increases almost by geometrical progression. Because an immediate duplication of 3 identical situations on the playing board ends a game in stalemate (both players repeat the same move 3 times in a row), the same as in a situation where the player is unable to make a move because of an imminent danger (check mate) and the number of figures is gradually decreasing, the total number of moves in each game is limited. Dixit and others (2009, 66) state that the total number of moves does not exceed 10^{120}.

An important feature of this game is a sequence of steps and thus the decision process in each move assumes a change of strategy based upon the change of situation on the playing board. Players can't (thoroughly) use symmetric strategies. This is a result of a succession of moves and it gives a player with white figures (who starts the game) a certain advantage. His advantage stems from the fact that with his choice he changes the alternatives for an opponent's possible moves (he must adjust to the move) and pre–determines suitable strategies of both players in subsequent steps. Based on the prediction of future development the first player determines possible situations on the playing board. In this case we talk about the advantage of the first move. The advantage of the second move lies in the possibility to react to the opponent's move (Nim game). With respect to the almost unlimited number of possible counter moves and the succession of an opponent's choice — no matter what strategy the first player chooses — the opponent can pick a counter strategy, which can lead to victory.

According to *Zermelo's axiom* (let's say that T is any set of outcomes in a finite game of two players with perfect information without accidental moves. Then, either player 1 can secure an outcome from T, or player 2 can secure an outcome from non T. — compare Hykšová, 2012, 73) one of the players must win, namely the one who chooses the best of all possible (constantly shrinking) options of right moves.

Since chess is a game, the main motive is to win. The victory, however, will not be reached merely by realization of our own choices, but also via predicting and considering both the potential and the actual future moves of the opponent. It means that the success of a player hinges also on the moves of the other player. These can't be fully influenced (besides a situation when he has no other move left) and thus it is critical to predict them.

Unfortunately, the number of player's and opponent's choices is excessive and thus their complete assessment is not realistic. (A computer processing a quadrillion operations per second would take 10^{100}years to compute it, but unfortunately the astronomers are predicting the demise of the Sun in 10^{10} years. Compare Dixit,

2009, 66) Hence it is important to select only the meaningful and the probable ones. In chess the point is to study possible, rational and probable moves. The problem lies in the fact that we can't see the opponent's moves in advance and the main strategy of a player is not to disclose his intentions so that they can't be thwarted. Chess can therefore possess certain characteristics of a game with a subjective assessment of risk as well as a version of a game with uncertainty.

3.2 Recommended literature

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4. Decisions under risk

Key words: *probability, minimax, maximax, Laplace Principle, Hurwitz's criteria, criterion of maximum regret*

The term *risk* is defined as the mental capacity of a player realizing the probability distribution of possible scenarios which come as a result of a certain choice. Risk can be valued subjectively, but can also convey a mathematical probability linked to the occurrence of certain consequences. Kolgomorov's probability calculation is a classic example of the mathematical axiomatic model of probability.

4.1 Philosophical concepts of probability

"*Classical interpretation, advocated by Laplace, Pascal, Bernoulli and Leibnitz, holds the probability of an event to be a fraction of the total number of possible ways in which the event can occur* " (Peterson, 2010, 134). This interpretation pre–supposes that all possible outcomes are equally likely. An example of this understanding of probability can be demonstrated via the game of Russian roulette played with one bullet in a six chamber revolver hand gun. We believe that the probability of the firing pin striking the bullet is the same as striking the empty chamber (1/6).

Another interpretation is *logical probability* (Carnap, Keynes). It takes into consideration not only the number of all possible events, but also their relevance. The probability of certain states

occurring is not uniform, but it's loaded by available observations proving the occurrence of a certain status (divorce).

Frequency interpretation holds that the probability of an event is a ratio between the number of times the event has occurred divided by the total number of observed cases. The most significant example of this understanding of probability is Hume's concept of probability. Inductive comprehension of probability increases its reliability with the growing number of performed observations (white swan).

Tendency interpretation is preferred by Popper's approach and it characterizes probability as an expression of an objective state of the world or a long–term tendency of things turning out a certain way. It is a longitudinal understanding or a statistical approach toward probability derived from Pierce's pragmatism (a coin falling on heads or tails).

Subjective interpretation diverges from the objective state of the world. Instead, it sees probability as a degree of certainty showing the validity of our convictions. Bayesian logic, derived from the teachings of Franck Ramsay and Thomas Bayes, is an example of this approach. Because decisions under uncertainty are based mainly on a subjective perception of probability, decision processes based on a Bayesian concept of probability are part of a special chapter of GT.

A diverse view of probability leads to using different approaches in various situations. Some are relatively simple, but not quite right. Some are more accurate, but their application in common situations is too complicated. David Lewis therefore came up with the *concept of the best system of probability* (or the most economical system), which leads to *"optimal balance between simplicity and information value"* (Beisbart, Harmann, 2011, 18).

4.2 Decisions under risk

A lottery is the most common example of decision under risk. In a lottery the chance of winning is (based on the classical theory of probability) determined by a total number of possible winnings or lottery tickets. Thus, it seems that we are dealing with a purely mathematical function of probability where we are taking into consideration only three variables — the investment, the amount of payoff and the probability of winning. Daniel Bernoulli supposed that a motive for entering a lottery derives from utility theory rather than the theory of probability. The reason is that with each extra ticket I buy my investments grow linearly, but the chances of winning grow in inverse geometric proportion. In other words: I myself am lowering the chances of winning for each extra bought ticket as well as the amount of possible payoff. The median utility says that the willingness to enter a game rises with the increase in the amount of possible winnings compared to an already existing fortune. (In other words, the bigger the amount of potential payoff in proportion to my existing fortune, the bigger my willingness to participate in a game.) The same game may not motivate a rich person as it might a poor one.

Gabriel Cramer, however, realized that while assessing utility we often encounter certain limitations — possible winnings are not perceived in the same manner as possible losses and not all people are equally sensitive to monetary outcomes. Furthermore, there is a mathematically determinable point which wipes out the difference between the median utility of people with different fortunes — with high values like 2^{24} we are unable to see an increase in utility (Hykšová, 2012, 25). This can be demonstrated in the St. Petersburg paradox. It shows that if we were to gain a geometrically growing payoff in a game of tossing the coin depending on whether the head falls in the first or the n^{th} toss (2^2, 2^n), we would see that the potential payoff grows almost infinitely, but our willingness to continue in the game falls with the increasing value of n, because the chances of gaining the prize are also falling.

Bernoulli's concept of lottery is thus tied to expected value and doesn't make provisions for its relativity toward the initial status and for possible losses. Equally, it doesn't take into consideration

the affiliations of certain players to profits or aversion to losses. According to Amos Tversky's and Daniel Kahneman's *Prospect theory,* which is often considered to be the foundation of behavioral economy, people demonstrate greater aversion to risk than affiliation to profit, preference of certainty (status quo) over uncertain profits as well as framing of information (received about a game) and unequal assessment of this information (Kahneman, Tversky, 1979, 263 — 291). It also doesn't reflect the total dominance of potential negative outcome (death) in games with different types of outcomes (such as Russian roulette).

However, in principle we can say that despite knowing the probability distribution of profits and losses, in games under risk players differentiate their strategies according to pursued preferences similarly to decisions under uncertainty (expected value of payoff, expected value of losses, chances of reaching the acceptable level).

4.3 Decisions under uncertainty

> Decisions under uncertainty can be characterized by the following conditions: either the exact probability distribution of profits or losses is unknown or it is irrelevant. In this situation, we try to maximize our chances of success or eliminate relevant risks.

The easiest way to contemplate the occurrence of unknown, unwanted consequences is to use the Bernoulli–Laplace principle. It supposes that an ideal strategy is one that chooses the maximum median utility based on an assumption that the occurrence of possible consequences happens with equal probability. In other words: an optimal strategy brings the highest effect in a situation with equal probability of occurrence of a certain consequence and thus turns decisions under uncertainty into decisions under equal risk.

When using this strategy, we weigh only the value of possible profits and losses irrespective of real frequency of their occurrence.

Since in real life the occurrence of certain consequences may not be distributed equally, in our attempt to avert unwanted consequences we can use *maximin (pessimistic) strategy*, which supposes greater occurrence of risks than profits and which manifests itself in a greater aversion to risk. Maximin strategy (Wald's criterion) ranks strategies on the basis of their worst–case outcomes (based on their minimum utility) and chooses one that best eliminates losses (the level of minimums is at its maximum). With this strategy, we are not trying to maximize our profits but minimize possible losses, which can be a way of reaching total gains.

Maximax strategy is the opposite process. It aims at gaining maximum profit and supposes that the probability of gaining it is higher than the probability of loss, or that possible gains exceed possible losses. *Maximax (optimistic) strategy* rates the utility of possible consequences of certain strategies and chooses one that leads to maximum possible profit (maximum of maximums). The use of this strategy allows the winner to gain the jackpot in the shortest possible time while undertaking a risk that other–than–predicted distribution might lead to loss.

Hurwitz's criterion is a combination of pessimistic and optimistic approaches. It attempts to eliminate both the exaggeratedly optimistic as well as the unrealistically pessimistic assessments. Instead of median utility, it chooses a weighted average of the smallest (maximin) and the biggest (maximax) utility, thereby making provisions for both the possible minimal profit (loss) as well as the maximum winnings, based on their own weighted significance.

Savage's criterion of maximum loss is a different approach. While the previously mentioned procedures pay attention mainly to incurred consequences, Savage proposes to concentrate on the differences between the actually achieved and the maximum potential outcomes. The determining criterion of this approach is to find a strategy which is able in the best possible way to eliminate

any payoff differences between the best possible outcome and the actual outcome (regret). This approach can be best utilized in repeated games as it aims at minimizing regrets, which, in fact, is a modification of Wald's maximin approach when dominantly using maximax strategy.

4.4 Recommended literature

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5. Games in normal (strategic) form

Key words: *normal pattern, dominance, equilibrium state, Nash, Pareto*

A lottery is an example of a game with risk against Nature. Another example is a game against Nature with certainty. But also games of two or more players are characterized by uncertainty and risk, especially in cases of normal form games.

> *A normal form game* is a type of game in which a player makes decisions independent of choices of his opponent (or opponents) while other players also make decisions independent of the choices of the first player. An important element of making decisions is that players make them simultaneously and they are not informed about each other's choices.

Perhaps the most common and best analyzed type of non–cooperative normal game is the prisoner's dilemma.

5.1 Prisoner's dilemma

The prisoner's dilemma (PD) indicates a paradigmatic example of a game with uncertainty and with non–zero sum of payoffs. Its essence is to find an optimal strategy eliminating risks stemming

from the inability to foresee an opponent's decisions (uncertainty) while the opponent's decisions directly influence the amount of the first player's payoff.

The history of PD goes all the way back to the year 1950, when Merrill Flood and Melwin Dresher (both from a company called RAND) created a mental experiment describing rational behavior of players who, independent of each other, pursue their own interests. Subsequently Albert W. Tucker — a mathematician at Princeton University and John Nash's teacher — formalized Dresher's experiment and came up with a story after which the dilemma has been named (Poundstone, 1992, 101 — 119). In the original formulation the problem is as follows:

"*Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, the police offer each prisoner a Faustian bargain. If he testifies against his partner, he will go free while the partner will get three years in prison on the main charge. Both of them are informed that his partner received the same offer. If both prisoners testify against each other, both will be sentenced to two years in jail. The prisoners are allowed some time for contemplating their decisions, however in no way can they learn of each other's decisions*" (Poundstone, 1992, 118).

What should the prisoners do? As they are unable to influence each other's decision, they should take care of their own well–being.

5.2 Table of possible solutions

When taking no account of social, moral and psychological aspects of the problem (potential revenge, reaction of people around, etc.) and when dealing only with purely rational decisions of players, we arrive at something which is called the table of possible solutions.

If player no. 1 decides to stay loyal to his accomplice, he doesn't defect and the punishment totally depends on the decision of the second player. If this player exhibits an equal level of loyalty and doesn't betray the first one, we see a mutual cooperation, the result of which is a year in prison for both. If the second player doesn't cooperate and betrays, the first player's outcome is a 3 year prison sentence. To cooperate, therefore, leads to certainty of imprisonment, for either 1 or 3 years.

If player 1 decides not to cooperate and betrays his accomplice his punishment again depends on the behavior of the second player. If he too prefers betrayal, the outcome is a two year sentence for both. But if player 2 chooses to cooperate and refuses to testify, the outcome for the first player is freedom. Player 1 can thus, via non–cooperation, achieve either 2 years in prison or freedom.

5.3 Dominant strategy

As the players are unable to communicate and decide a common strategy, each rational player chooses a strategy which will lead, whatever the choices of the other player might be, to maximum profit and optimum solution. This strategy is called *dominant strategy*. In dominant strategy a player always achieves the best possible results regardless of an opponent's decisions.

This goes to show that for player 1 a better solution is not to cooperate. Non–cooperation leads to 2 years in prison or freedom while cooperation (not testifying) results in certain imprisonment for either one or three years. As far as the 1st player is concerned, this is a *strictly dominant strategy* despite the fact that there is a solution leading to a worse result when not cooperating (2 years in prison) than in a situation of mutual cooperation (1 year). From the 1st player's point of view, this strategy leads to better results than are potential results of the 2nd player. If player 1 doesn't cooperate, he gains freedom or 2 years in prison while his opponent receives 2 or 3 years in prison.

In a situation in which consequences of certain decisions lead to worse or at least the same result as when using a different strategy, we talk about *weak dominance*. The worse strategy is dominated by the better one and the better one is dominant.

Osbourne's model with intersection is a good example of the difference between strict and weak dominance (Osbourne, 2004, 1). Let's assume we are entering an intersection. There is a car standing in the right lane (which can travel straight or turn right, but is letting some pedestrians cross the road). The left lane is free. In this situation it is preferable to choose the left lane (strict dominance). However, if there is a car also in the left lane, which can travel straight, the choice of the left lane is still better although the choice of the right lane is only slightly dominated because it either leads to a worse choice (delay caused by turning right) or to an equally good choice (of going straight).

5.4 Nash equilibrium

Dominant strategies express optimal solutions of situations for individual players. In PD, the same strategy is optimal for both players. In mutual application of dominant strategies we see an equilibrium in which no player can gain an advantage by one–sidedly changing a strategy (Nash equilibrium). John Forbes Nash proved that each finite game has at least one such solution and we call these states equilibriums.

5.5 Pareto optimum

From the nature of PD we can conclude that the application of non–cooperative strategy is preferable for both accused with respect to elimination of an opponent's moves, but it doesn't always lead to the best result (freedom). Equally rational players will eventually end up in a situation when both testify and thus get two year prison sentences, which is worse than in a situation of cooperation.

And thus equilibrium doesn't necessarily lead to Pareto optimum. Pareto's optimal solution is an equilibrium point — if a player wants to improve his situation he can do so only at the expense of the other player. Pareto optimum is a perfect equilibrium.

PD, however, might involve not only two players. The number of players simultaneously playing a game can be *n*. The process of decision–making in multi–personal games does not necessarily make searching for optimal strategy more difficult, but it makes it harder to find Pareto optimum. In different socio–economic games we can have more than one equilibrium point.

Some non–cooperative games can be solved by repeatedly eliminating strictly dominated strategies (what's left is a dominant strategy), but for others this does not apply. Spousal dispute (war of the sexes) is a good example of this type of game. In it, both spouses manifest different preferences for spending their free time. The husband prefers sport while the wife is keen on ballet. Neither of them identifies with the preferences of the other. The possibility of spending time together means sacrifice for one or the other. Spending free time separately would rip them apart and finding a compromise is impossible. In this scenario the preferences are distributed symmetrically, but possible outcomes are asymmetric. Both partners realize that spending an evening together is possible only when one is completely dominating while the other's outcome is a complete loss. Subordinating oneself to one's partner (cooperation), however, results in a higher profit than splitting of the couple despite the asymmetric utility. (A similar example is the Starbucks and a local café, Comp. Dixit, et al., 2009, 115)

The coward's dilemma is a different example. The main point is sticking to the chosen strategy while undertaking a risk that the same strategy of the opponent would lead to a total loss for both players. Howard Raiffa describes it in an example of two people driving in the same lane straight towards each other. (Originally unpublished, appeared in Osbourne, Rubinstein, 1994, 30). Whoever swerves is considered a 'chicken' and loses, the other one is a

'hawk' (a different version of this game is the hawk–dove dilemma). The problem is that in a game without any information about the intention of the other player, none of the players has a dominant strategy because in both situations, both players swerve or none of them swerve, lead to a loss. Repeating this game doesn't change the situation as it enables to uncover the tacit common knowledge or it helps create a platform for coordinated and cooperative behavior.

5.6 Recommended literature

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6. Iterated games

Key words: *finite iteration, infinite iteration, sanctions, (a)symmetry, leadership*

When finding an optimal strategy in classic PD, rational subjects are forced to eliminate the influence of a possible opponent's retaliation. In iterated games the retaliation and "reading the opponent's strategy" is the key point of finding the best tactics.

In one–off PD, players find the equilibrium when using the "traitor" strategy. But repeating this strategy doesn't bring the desired benefit, because a) even if it had led to maximum profit in the first step, we can't expect the same opponent to choose the same self–destructive behavior again (if he is aware of his opponent's previous move or he is interested in reducing the losses from the previous step), b) the total benefit of both players is not at its maximum. By choosing the traitor strategy, the sum of granted sentences is either 3 years (with an irrational player) or 4 years (with rational players), which is more than the total of 2 years (1 year for each player) in a situation of cooperation. The cooperation can't be formally agreed upon. However, in iterated games rational players in their quest for the highest possible personal benefit choose strategies which are similar to cooperation that might have been negotiated.

There are several factors which play an important role in iterated games: whether the players know the opponent's choice, the number of iterations, the number of opponents and the diversity of chosen anti–strategies or the number of games left.

If the player isn't informed about the opponent's reaction, we talk about classic PD. If he is aware of it, he adjusts his strategy according to an opponent's behavior or according to the profitability of his strategies in previous moves.

6.1 Games with definite and indefinite number of iterations

Based on the number of iterations, games can be divided into two categories: *games with a definite number of iteration* and *games with an indefinite number of iterations*. In principle we can think of all games as games with a finite number of repeats. All games must end. If not for other reasons, then simply due to the finality of a player's existence (unless we are talking about a game with two infinitely lasting players). Mortality (finality) of a player, however, can be considered irrelevant in situations, in which he is replaced by other player(s) or when the number of iterations during his existence is potentially so big (possible social interactions) that there is no point in considering such a number (Osborne, Rubinstein, 1994, 135).

More important than the number of iterations is how the final horizon is perceived by players. If players clearly feel the proximity of the game's end, then the closer they are to it the more their choices approximate the choice of classic PD without iterations. In games with a very small number of repeats or right before the end of the game, players aren't motivated to cooperate and choose the dominant strategy of betrayal. In games with an infinite or *indefinite number of repeats* the players with each consecutive repeat get a stronger conviction that the game will continue and thus adjust their strategies accordingly (Osborne, Rubinstein, 1994, 135). Strategy which takes into consideration the success of previous moves and a prior opponent's behavior is called *contingent strategy*.

In general we can say that in games with a finite number of iteration there is at least one Nash equilibrium point which characterizes the optimal strategy of decision–making, whilst in games with an infinite number of repeats we can expect a wider palette of equilibrium points based upon chosen strategies. Axelrod's tournament of different algorithms used in PD is a good example of the difference in profitability of various strategies in iterated games.

6.2 Axelrod's tournament

In 1981 Robert Axelrod and the RAND company asked 15 theorists of GT to submit various strategies of PD for a tournament. In this tournament each strategy played against each other (15x15) in a game of 200 moves (iterations). The tournament consisted of differently built algorithms, which Axelrod divided based on 4 criteria: to be nice, to be provocative, not to be envious, not to be too clever.

6.3 Classification of strategies

Helpful strategies were those which didn't start with a betrayal, but by an offer for cooperation. *Wicked strategies* are their opposites.

Forgiving strategies didn't react to an opponent's betrayal at all or only partially while *unforgiving strategies* reacted to it immediately, constantly and thoroughly.

An envious strategy preferred an opponent's loss to one's own victory and therefore it concentrated on minimizing an opponent's gains. In this situation PD is thought of as an antagonistic game. It's opposite is *non–envious strategy*, which concentrates on maximizing one's own payoffs independent of an opponent's gains.

Clever (cunning, probing) strategies were based on occasional betrayals with a mostly cooperative approach. *Non–clever* strategies rigorously followed either a cooperative or non–cooperative mode.

Axelrod's tournament consisted of various approaches: *always cooperate, always defects, spiteful approach* (cooperates till first betrayal, after which it doesn't cooperate), *tit for tat* (friendly copying of the opponent), *mistrust* (unfriendly tit for tat), *naive prober* (occasionally defects), *remorseful* prober (reacts to betrayal by cooperation), *hard tit for tat* (doesn't cooperate if opponent defected in two out of the last three moves), *gradual tit for tat* (with each following betrayal increasing the frequency of non–cooperation), *gradual killer* (at first doesn't cooperate and then reciprocates an opponent's behavior in the 6th and 7th move), *hard tit for two tats* (no cooperation if there were 2 betrayals in 3 moves), *soft tit for two tats* (no cooperation after 2 consecutive betrayals), *slow tit for tat* (revenge with delay), *periodical defect–defect–cooperation, periodical cooperation–cooperation–defect, soft majority* (cooperation according to a majority decision of the opponent), *hard majority* (non–cooperation according to a majority decision of the opponent), *Pavlov* (cooperates if there was a match in a previous move), *Pavlov Px* (adjusting defect according to the previous average), *random, hard Joss* (tit for tat + defect with a probability of 0.9), *soft Joss* (tit for tat + cooperation with a probability of 0.9), *better and better* (cooperation grows with the number of moves), *worse and worse* (betrayal grows with the number of moves) — The list above also contains later modifications of the original 15 strategies (compare Hykšová, 2012, 214 — 217).

The tournament's outcome showed that friendly strategies, especially *Tit for tat*, received the highest scores. Clever and unfriendly strategies scored against naively friendly algorithms, but when used simultaneously, they mutually eliminated the possibility of gaining maximum payoffs, which made them less successful than friendly and forgiving strategies. After publication of results and modification of used algorithms, Axelrod's test was repeated with 1000 iterations. Rappaport's friendly tit for tat strategy repeatedly proved to be the most successful. Michael Nowak and Karol Sigmund (Démuth, 2009, 182) later elaborated this strategy into the Pavlov algorithm (Nowak, Sigmund, 1993, 56 — 58, for further development read: Grossman, 2004)

Tit for tat strategy and its modifications prove a higher effectiveness and rationality of friendly pseudo–cooperating strategies of two or more players compared to one–off non–cooperative strategies.

The total choice of optimal strategy in iterated games depends (apart from the multitude and variability of anti–strategies) also on the amount of winnings and losses and on how symmetrical or asymmetrical the relationships are.

In simple PD we can disregard the secondary payoffs connected to certain choices. Betrayal of the accomplice can lead to avoiding prison, but at the same time it can result in punishment from the accomplice or part of society. This punishment (death, violence…) can, however, be worse than the impending sanction resulting from a chosen strategy. This changes an iterated PD into a game with multi–criteria decision–making while punishments and rewards become the central motive for pro–cooperative behavior and for creation of such social rules which lead to preference of cooperation over non–cooperation.

Another important criterion is the *symmetry of players' significance*. It is quite understandable that the willingness to play a game is directly proportional to the ratio of potential profit to initial capital. It follows that unequally "strong" players would be unequally motivated to undergo the same risk with the same potential payoff.

6.4. Leadership

A good example of this is *leadership*. If two equally large countries are threatened by an epidemic with the same ratio of mortality, neither of these countries would be willing to solely finance the vaccine research that would also benefit the other country. On the other hand, if the difference in population is so large that the

mortality in one country would cause a decrease of GDP, larger than the cost of the research, the larger country would voluntarily carry out the research, while the smaller country would not be expected to do so (Dixit, 2009, 412). A similar solution of global conflicts can be expected mainly from those, who in absolute terms gain the most from resolving these conflicts. Asymmetry of players' significance (size, power, etc.) is just one of the problems in PD. Another example *is information asymmetry*. In many games and especially in economic behavior, unequal access to information is a common occurrence. A player with limited information is at a disadvantage and therefore chooses eliminating strategies, while a player with sufficient information or with perfect information can choose a strategy leading to optimization of potential payoffs. Iterated games lead to conscious or unconscious cooperation. Martin Nowak formulated *five basic rules of cooperative behavior: kin selection, direct reciprocity, indirect reciprocity, network reciprocity and group selection* (Nowak, 2006, 1560 — 1563). The last four emerge from the rationality of infinite games.

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7. Cooperative games

Key words: *monopoly, duopoly, oligopoly, cooperative games, bargaining*

Iterated games draw the attention to the difference between a player's dominant strategy and optimal payoffs of individual decision–makers. Gaining optimal payoffs in iterated games is the cornerstone of ideas of many economists of the 19th century. In 1838, Antoine Augustine Cournot published a work analyzing profit optimization in different types of economic competitions.

7.1 Monopoly

The simplest form of competition is a competition with a single player in the market — *monopoly*, which resembles a game against nature. Since in monopoly there is only one producer and his payoffs more or less depend only on the product's price and the number of sold pieces, it may seem that there isn't really any game. This would hold true if the price didn't influence the number of sold products. But in common situations it does, therefore the producer must optimize potential payoffs by setting the right price and optimal production quantities. Important variables in this calculation are the number of products which would have sold if the price had been different and the optimization of costs. Cournot calculated that the producer will gain maximum payoff $(1/2(M–c))^2$ if he floods the market with the maximum salable number of products

q, while considering the quantity of products and the price on the market *M* and production costs per item *c* (Hrubý, 2011, 6). Production costs may include all kinds of inputs — wages, material, equipment, but also different fees and taxes, which can lead to various results in different non–linear conditions in the market.

7.2 Duopoly

If there are two suppliers on the market with a relevant market share, the game changes into a *duopoly* — a variant of PD, since each supplier can influence his own profits by increasing the market share through reduction of prices. If the opponent doesn't change his strategy, the player with lower prices gradually forces the player with higher prices out of the market and thus attains a monopoly position. To avoid this, the second player modifies the price of his product to an equal or lower level than the price of the first player's product. After a series of similar steps, the players eventually reach an equilibrium, in which they gain lower profits than a monopoly player, but their total sales are higher. The drop in total gains caused by a lower unit price and higher production amount can lead duopoly players to optimization of their total gains by entering into a cartel agreement. This in fact changes PD into a monopoly with one major target: trying to find a maximum profit, which would then be divided between both players. From a customer's point of view, this is basically a monopoly of several producers. Consumers (or other potential players) are naturally trying to force a change. In case of another player trying to enter the market, this de–facto monopoly position is also quite unstable. As Joseph Louis Bertrand (1883) showed, it is advantageous for each player to turn away from the agreement by gaining higher market share as long as the lowering of prices and ramping–up sales leads to higher profits. This may eventually lead the competitors to finding equilibrium, which would actually be a production with zero profit.

7.3 Oligopoly

Bertrand's duopoly corresponds with an optimal economic situation, which according to Cournot is *oligopoly* or a game with an almost unlimited number of players without dominant market position. Oligopoly prevents cartel agreements and further pushes the competitors to minimize unit prices and increase market share. So called *perfect competition* is an ultimate case of oligopoly. It includes n players minimizing their profit all the way to zero, which in fact results in flooding the market with the maximum possible number of products selling for the lowest possible price. This sort of situation is ideal for a customer, but in the long run, would not be interesting for players. It is therefore just an ideal model without any application in a real economic situation, although in limited circumstances we can observe sales of products with a price lower than real expenses (e.g. dumping prices), which makes sense as a means of bettering one's market position (e.g. advertising, pushing out the competition) or as a way of reaching other targets in a different game (employment support, etc.).

Heinrich von Stackelberg's duopoly model is another solution for iterated games. It supposes division of potential profits in situations, in which the second player chooses his strategy according to the first player's strategy of choice (*the leader — follower rule*). The fact that the leader in advance (ex ante) presumes that the follower is watching him and is adjusting his strategy to the leader's choice is an essential factor of this model. With his decision the leader can cover a dominant part of the market while the follower can exploit the situation by joining the market with a smaller share. In Stackelberg's model, the behavior of the follower is not agreed or enforceable and thus it is a non–cooperative or quasi–cooperative game.

All the above mentioned examples (apart from the cartel agreements) prove the advantage of quasi–cooperation over purely egoid strategy. A special example of profit optimization is a cooperative game with clearly set strategies of players' behavior with respect to maximizing the total sum of payoffs in a given game.

7.4 Distribution of winnings

Based on winning distribution, cooperative games can be divided into two groups. The first group is *games with non–transferable winnings*. In these games winnings are tied to individual winners and their utility can't be transferred to other players. As an example of this type of game, we can mention supporting a candidate in a contest, which has a limited number of winners (e.g. presidential elections). Supporting a different candidate doesn't bring direct benefit to other participating subjects (only one candidate can become the president), but the result of this action might be better than any result without coalition support.

A second group of games are *games with transferrable winnings*. In these games the formal winner can redistribute the winnings among all other coalition members in a different ratio than was the ratio of their own winnings. An example of this type of game is the economic behavior of firms in cartels or consortiums. The profits from sales of individual products don't have to be distributed according to brands, but based on the effort invested into production and distribution of these products. Players can agree that one of them would specialize in the production of a certain product or component while the other might specialize in finalization or sale, but the total gains would be divided, for example, evenly. Some games can have a mixed character — both transferrable and non–transferable. Riders in the Tour de France break–out cooperate and collect points in hill or sprint awards (non–transferrable — possible gains only in the future) and exert their energy independently of potential win in the group in regard to the possible lead (advance) over the riders in the main peloton (transferability of finances for placement). An important factor supporting cooperative behavior is the possibility that by cooperation, players increase their winnings or at least the probability of achieving the winnings when compared to other — non–cooperative strategies.

In his work *The Bargaining Problem* John Nash formulated axioms encouraging players to enter into cooperative agreements. Providing 1/ the players (bargainers) are rational he supposed that 2/ the strategy they agree on corresponds with Nash's equilibrium. 3/ This equilibrium has the characteristics of Pareto's equilibrium and 4/ the agreed strategy is accepted only if according to players it leads to achievable targets with a high degree of probability. 5/ Chosen strategies (independent of their form) lead to the same preferred utility. 6/ The outcome is independent of irrelevant alternatives. 7/ An increase in utility in cooperative game is symmetrical for all players (strategies are identical only if their utility functions are identical — Peterson, 2010, 250, Hykšová, 2012, 310 — 313).

Ehud Kalai and Meir Smorodinsky modified Nash's axioms, points 5 — 7, (Kalai–Smorodinsky axiom) by substituting the independence of irrelevant alternatives with *a monotonicity condition*, which supposes that all profits stemming from cooperative games are better than those stemming from non–cooperation — *substantive game* or *Rubinstein bargaining model*, which prefers (infinite) alternating of offers (Rubinstein, 1982, 97 — 109).

Based on winning distribution we can distinguish between *guaranteed winnings* and mutual winnings. Guaranteed winnings are those which the player achieves via his own decision regardless of the other player's behavior. Mutual winnings are achieved through mutual cooperation. The *core* of the game is an aggregate of acceptable winnings distributions, which can be divided based on the principle of *charity distribution* (evenly), *fair distribution* (according to effort — Shapley index), *guaranteed payoffs + evenly* (Brožová, 2007, 78).

7.5 Recommended literature

NASH, J.: The bargaining problem. In: KUHN, H. W. (ed.): *Classics in Game Theory.* Princeton: Princeton University Press 1997, 5 — 13.

BINMORE, K.: *Game Theory and the Social Contract*. Volume 2. Just Playing. MIT Press 1998, 88 — 95.

8. Game theory and economic behavior

Key words: *free market, mechanisms design theory, tragedy of the commons, regulation of common sources*

Just the name of Neumann's and Morgenstern's work alone, but also the entire history of GT suggest that GT is closely tied to the description and rationalization of economic behavior. The speculations about rationality and computability of economic behavior are, however, much older and can be seen in the fundamentals of classic philosophical economics, (Sedláček, 2012). It is linked with mercantilism (it's criticism) and physiocratic thoughts (and their refusal), which formed the philosophical–economic opinions of Adam Smith.

8.1 Free market

The historical roots of GT date back to ancient texts, but the most famous formulation of GT and the economic consequences of the free market are contained mainly in the work of Bernard Mandeville called *The Fable of the Bees, or Private Vices, Public Benefits* (1704). However, the person who is usually considered to be the ideologist of the free market is Adam Smith.

Smith, following the ideas of physiocrats and the Laissez–faire movement, understood that the rationality of Nature isn't visible only in physical relations, but that Nature can be perceived as a prominent participant in wider socio–economic relations, and even (through its presence in our own nature) as an important player of social and economic games. According to him the world and Nature are created in a way, in which they create conditions for maximization of well–being and happiness of each individual. Desire for happiness and well–being (which are naturally common to all rational beings) and physical presence in the same environment change social–economic relationships into a game of maximization of happiness and well–being.

Smith was convinced that all of us pursue only our own egoistic goals. By fulfilling these goals we enter into a competition conflict, which at first sight renders general well–being impossible, but in the end it's the competition struggle that (as a non–cooperative game) improves conditions for all participating players.

The basic idea of Smith's work *An Inquiry into the Nature and Causes of the Wealth of Nations* is the image of *free competition* and *the invisible hand of the market*. If more contestants (buyers) show interest in the same goods (leading to their feeling of well–being) its price goes up or it becomes scarce. This can be perceived as a worsening of player's conditions on the side of the buyers. Rising prices and the lack of a certain commodity on the market, however, cause producers to see an opportunity for increasing their own profits (improving sellers' conditions) and thus are motivated to ramp–up production quantities by which, in the end, eliminate the scarcity of the goods. This invisible hand of the market creates permanent pressure to balance supply and demand. The surplus of goods results in falling prices (all the way down to expenses), which works in favor of other participants (buyers). Competition between producers works in a similar way. This is a fight for customers and thus a fight for higher profit.

One of the consequences of Smith's theory is that the competition between customers improves conditions of producers while at the same time the economic tension between producers positively influences a customer's position. Fulfillment of egoistic goals by members of a certain group, therefore, (paradoxically) leads to well–being enhancement of the members of the other group and vice versa.

When an individual desires the best and the cheapest product, he eventually and indirectly (providing the competition among producers exists) creates conditions for price cuts, which benefits all other buyers. Smith's theory (from a strategies point of view) thus presupposes the existence of a Nash equilibrium analogy and that is the balance between supply and demand and equally presupposes the existence of dominant non–cooperative strategies of members of the same group (customers or producers) leading to optimization of one's own well–being. Smith's market regulated by Nature is in fact a non–cooperative game between customers and producers, but also between customers and producers themselves. At the same time it is crucial that the market is free and unlimited. Smith had realized that any restrictions imposed on the market and pricing (customs, limits, subsidies, etc.) would result in deformation of the natural balance. Openness of the market is the only tool for elimination of cartels and cartel agreements. These are feasible only in markets where economic or legislative restrictions prevent other subjects from entering the game.

Thomas Malthusius and David Ricardo developed Smith's theory and created the classic economy model, which (along with Keynes theory) is one of the cornerstones of economic liberalism and free market capitalism.

8.2 Neumann and Morgenstern GT

In the thirties of the 20th century, many economic theories were turned upside down. After the global financial crises had set in, which led to World War II., it was necessary to re–evaluate classic theories and take into greater account certain indefinite and unknown variables, which impacted social and economic relations. Von Neumann and Morgenstern formulated a mathematical model which made provisions for the amount of risk involved and for decision–making with incomplete information, by which they formalized the process of economic decision–making while at the same time included risk into the calculation of total utility. GT thus offered a new tool for optimization of potential payoffs in various types of social–economic relations. The equilibrium analysis in the theory of non–cooperative games, awarded the Nobel Prize for economy in 1994 (John Nash, Reinhart Selten, John Harsanyi) is a notable contribution of GT. In 2007 GT was again awarded the Nobel Prize for economy, this time the prize went to Leonid Hurwitz, Eric Maskin and Roger Myerson for the mechanism design theory, which examines the influence of certain stimuli on economic behavior and searches for mechanisms designed to attain predetermined returns. Mechanism design theory helps differentiate between well and badly functioning markets and optimize players' direction in order to achieve a desired outcome.

GT and Smith's theory of market's invisible hand reveals how players and the market (in ideal conditions) secure maximal gain and effective allocation of precious sources. In reality, however, the conditions are rarely perfect and therefore deformations of the free market draw significant attention of many theorists. One of them is Garrett Hardin's mind experiment named *the tragedy of the commons*.

8.3 The tragedy of the Commons

Garett Hardin pointed out that in situations of limited resources, dominant strategies can lead to gradual depletion of potential payoffs and to total destruction of the market. As an example for his idea he chose a metaphor involving a common pasture shared by local cow herders. To the owner or the shepherd each cow brings certain utility in the form of milk or meat. Each rational owner is interested in maximizing the number of cows grazing on the commons. The problem of the commons is its restricted space and limited regeneration capability. In a certain clearly observable moment the pasture stops providing enough resources for all the cattle and the utility per cow starts going down. The owner can eliminate the decline in profit per cow by increasing the number of animals. The decline in profit per animal applies to all owners and this situation to a certain extent can still be effective to an egoistic owner (profit per animal is lower, total payoff from all cattle is higher). Individual profits are higher than costs since the costs are shared (*Commonize Cost — Privatize Profits*). This results in a general ongoing motivation to add more and more animals to the herd.

However, after exceeding a certain critical number the pasture is depleted due to overgrazing, which as a result leads to a total collapse of the cattle breeding and thus of the entire market.

8.4 Common resources governance

Based on the previous example, many economists came to the conclusion that some sort of market regulation is needed, especially in markets with common sources. The solution preferred by Hardin leads to fencing, privatization and subsequent protection of sources, although he admits that not all sources can be private. This holds true mainly for water (drinking water as well as oceans and fishing waters), air, natural resources as well as space. Elinor Ostrom objected to Hardin's solutions and postulated a framework of measures necessary for common sources governance. As a result of her work, she (along with Oliver E. Williams) received the Nobel Prize in Economics in 2009. The core of her research lies in her conviction that the tragedy of the commons is an example of limited rationality and that truly rational subjects would be able to arrive at a self–regulating system eliminating the dangers of boundless egoism while using the common pasture.

8.5 Recommended literature

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SEDLÁČEK, T.: *Economics of Good and Evil. The Quest for Economic Meaning from Gilgamesh to Wall Street*, Oxford University Press, 2011, 183 — 212.

9. Paradoxes and anomalies

Key words: *Russell's turkey, Newcomb paradox, Allais paradox, Ellsber paradox, Arrow paradox*

Repeated occurrence of financial and economic crisis, along with our inability to foresee the future of economic growth and the behavior of individuals, question the validity of mathematical models of GT as well as the purpose of economics itself. As Tomáš Sedláček states, if we believe in economic prophecies of crisis and we take them seriously (act according to them), negative consequences usually don't occur — *Jonah's paradox* (Sedláček, 2012, 319 — 320). Some science philosophers doubt the ability of economic science to attain the accuracy and reliability of nature (and exact) sciences and as a reason they state poor predictability of economic rules and laws.

9.1 Limited rationality

One of the most frequently cited reasons for inaccuracy of economic theories is their simplification and elimination of several important factors of decision–making. GT supposes that the participant in economic games is a *Dennett agent*, in other words a purely rational being. Humans, however, are not purely rational. More often than not, the opposite is true. As Antonio Damasio says: "*We aren't thinking systems that sometimes have emotions, we are emotional systems that sometimes think*" (Damasio, 2000, paraphrased by

Koukolík, 2003, 203). Economic behavior is influenced by many social, emotional or irrational moments, which can be documented for example by panic that spreads after the release of certain information, such as alleged insolvency of a bank, ominous stock–market regression or other mob behavior. Stock brokers could tell stories of emotional, aesthetic, or other strange reason of clients for preferring certain investments, but also of various phenomena, which are gradually becoming part of neuromarketing.

Limited rationality, "groupthink" (Koukolík, Drtilová, 2001, 2002) and the fact that we are not purely rational and our brain often becomes the slave of our passions are not the only reasons for imperfect predictability in economic theories. Nassim Nicholas Taleb (2011, 28, 59) states that we must search for other reasons in the methodology, on which we base our knowledge.

This methodology is for the most part built on our conviction that with the growing number of observations we penetrate deeper and deeper into the understanding of historical and causal laws of social–economic behavior.

9.2 Black swan

Taleb (similarly to K.R. Popper in his work *The Poverty of Historicism*) is convinced that the knowledge of history is subject to illusions of its understanding. It is possible only *ex post* and not *ex ante* because we determine the causal nexus by elimination of other influences. Another problem is in the inductive character of learning about economic behavior. By observing previous experiences we gradually get an idea about future events. This image, however, is based on observations of the most frequented and average occurrences and not on observations of all possible events. Thus, we are getting into the position of *Russell's turkey*. Based on everyday feeding, the turkey infers a conviction that his owner is interested in its utmost well–being. The certainty of this conviction grows with every feeding and is at its highest when the bird is in the greatest danger. The black swan, on the other hand, represents an idea that although they are quite rare, with long enough observation we are very likely to eventually find them.

Many science philosophers (Hollis, Sudgen, 1993, 1–35) maintain that economics and GT will never be able to claim certainty of scientific predictions as they are built on too many paradoxes. One of the best known paradoxes of GT is Newcomb's paradox.

9.3 GT and the knowledge of cognitive psychology

Robert Nozick introduced Newcomb's paradox in the form of a game. It is a variant of PD in which a player can choose from two alternatives — he either takes only box number 2 or takes both box number 1 and 2. An entity somehow presented as being exceptionally skilled at predicting people's actions (predictor) had placed $1000 into box number 1 and nothing into box number 2 (if the predictor supposes the player will choose both boxes) or $1 million if the predictor believes the player will choose only box number 2. What's the ideal strategy for each player? It is quite apparent that the first player would gain the highest payoff if he chooses only box number 2. Then he can win $1 million. However, since he is not informed about the predictor's behavior, he makes his decision under uncertainty or risk. Choosing both boxes will eliminate this risk. Then he can be sure of $1000, but at the same time (depending on the predictor's infallibility) loses a potential $1 million as a result of his unwillingness to risk. On the other hand, whatever the choice of the player is, the predicting ability of the opponent had determined the content of both boxes and the player's choice will not change it. Newcomb's paradox illustrates the problem of freedom

of choice and it's rationality in games with complete information.

Another paradox was formulated by Maurice Allais — winner of the Nobel Prize in Economics in 1988. In *Allais paradox*, a player can choose between A1: certain win of 1 million francs or A2: 10 percent probability of winning 5 million francs, 89 percent probability of winning 1 million francs and 1 percent probability of getting nothing. In the second choice, he can choose between B1: 11 percent probability of winning 1 million francs, 89 percent probability of winning nothing or a more risky variant B2: 10 percent probability of getting 5 million versus 90 percent probability of zero winnings. Based on the theory of expected utility, we should prefer more profitable alternatives, but the reality proves that players in the first variant prefer certainty over potentially high profits (A1>A2), but in alternative 2, the expected utility wins over certainty (B2>B1).

Ellsber's paradox represents a similar problem. Suppose you have an urn containing 30 red balls and 60 other balls that are either black or yellow. You don't know how many black or how many yellow balls there are, but that the total number of black balls plus the total number of yellow ones equals 60. If we favor choice V1, we receive $100 if we draw a red ball. If we draw a black one (V2), we also receive $100. Which choice should be favored?

In choice V3, we also receive $100 if red or yellow balls are drawn. But in choice V4, the prize is also $100, if we draw a black or yellow ball. It seems that logically speaking the above mentioned choices are paralogisms as there is no way of determining the exact probability of V2 and V3 and thus it is impossible to assess their profitability. Despite this, people prefer V1 and V4, as they want to avoid uncertainty (Savage's certainty principle) even if they may win less.

A similar problem is analyzed in *the paradox of two envelopes*. If you have to choose between two identical envelopes with different sums of money (A, 2A) you can do it randomly. Before you open the envelope, you are offered the chance to take the other envelope instead. Based on the theory of expected utility, the median value of the amount in the second envelope is 0,5 x 2A+0,5 x 0,5A=(5/4)A, which means it is worth swapping the envelope. This leads to the paradox that it is beneficial to continue to swap envelopes indefinitely. This paradox thus refers to *Buridan's donkey paradox*. It is similarly difficult to determine the winnings in the *St. Petersburg paradox* (chapter 4). The player in this game wins 2^{n-1}dollars if the coin is tossed n times until the first head appears. If theoretically the game is played infinitely, the winnings also grow infinitely while the success rate falls equally endlessly. Nobody, however, would be willing to pay the casino a fee for entering the game as the amount of this fee is based on the portion of possibly endless payoffs and thus the portion would also have to be infinite.

Arrow's paradox or *Arrow's impossibility theorem* is another logical paradox in GT (Kenneth Arrow — winner of the Nobel Prize in Economics in 1972, for his theory of social choice). This paradox describes a decision process of two or more players who are choosing between several options and the resulting impossibility of finding a mechanism for an effective solution. If different players have different preferences, a situation can arise in which neither of the offered alternatives will be acceptable more than the others. A vote of a pirate's captain (like in the movie Pirates of the Caribbean III), in which all pirates vote for themselves can serve as an example of this problem. Similarly impossible is a search for an optimal trash dumping site which is based on the principle of the longest distance from residences in an area with homogenous spacing of homes.

Zeckhauser's paradox calls attention to the relativity of payoffs in situations with different probabilities. Suppose, you are playing Russian roulette and there are two bullets in a six chamber revolver. A) What sum are you willing to pay for removing the bullets? B) What sum would you pay for removing a single bullet out of four in a six chamber revolver? Since the money is worthless after you are dead, it would be rational to sacrifice everything in both unequal situations. How can unequal situations lead to equal payoffs? Imagine that 1) there are two bullets in a six chamber gun. How much would you pay for their removal? 2) There is only one bullet in a three chamber gun. How much would you pay for removing it? 3) There is a 50 % probability you will be executed and a 50 % probability you will play Russian roulette with one bullet in a three chamber gun. How much are you willing to pay for removing this bullet? 4) What sum would you sacrifice for removing a single bullet out of four in a six chamber revolver?

From previous modification of Zeckhauser's paradox, we can ascertain that the probability of death in situations 1 and 2 is the same, therefore the price for elimination of the bullets should be equal. In situation 3, the probability of certain death is 50 %, but the second part of this case equals situation 2. Since the probability of execution can't be influenced, only the second part is meaningful and that is the same as in 2. It means that the price in 3 should be identical to situation 2. And since in the last situation three out of four bullets are non–negotiable, the probability of death is again 50 %. Buying one bullet makes the situation identical to situation 3, which is, in fact, an equivalent of 2 and the price in situation 2 is identical to situation 1, it follows that the price of the bullet in situation 4 should be identical to the price in 1.

Another paradox in game theory was described by Juan Parrondo (*Parrondo's paradox*). There exist pairs of games, each with a higher probability of losing than winning, for which it is possible to construct a winning strategy by playing the games alternately. *Easterlin's paradox* reveals minimal correlation between the level of society's economic development and an average level of happiness.

9.4 Recommended literature

PETERSON, M.: *An Introduction to Decision Theory*. Cambridge: Cambridge University Press, 2010, 78 — 90.

BINMORE, K.: *Rational decisions*. Princeton: Princeton University Press, 2009, 26 — 58.

TALEB, N., N.: *The Black Swan: The Impact of the Highly Improbable.* Random House 2010, 190 — 200.

10. GT and the theory of social contract

Key words: *utilitarianism, pro–social behavior, common good, comparability of utility, empathizing*

It can be said that despite possessing many mathematic tools, GT doesn't prove very successful in the area of macroeconomics (Application of its knowledge has been more successful in microeconomics. Compare Lacasse, Ross, 1994, 379 — 387). Moreover, as described in a previous chapter, it is full of antinomies and paradoxes. Despite this it provides us with a good tool for understanding and clarification of other — more global phenomena of human behavior, namely social and pro–social behavior.

10.1 Pro–social behavior as a manifestation of egoism

The conviction that social affection of humans has a deeper rational background is as old as philosophy itself, but the search for concrete reasons why people seek communities and behave socially began with Thomas Hobbes. He realized that while studying social behavior we discover that we are not social because we adore other people, but mainly because there is a greater advantage to do so. The existence of anti–social behavior, egoism, criminality, envy, libel and other social ailments as well as the simple reality that we don't feel the same affection to all people at all times and that we select our friends (not to mention our partners) are examples of the fact that pro–social behavior is not inherent. So why do we

like some people and seek their company while avoiding others?

Hobbes is convinced it is a result of simple social–economic calculating. We seek the presence of others if it serves us more than their absence. In a similar way, we select who to spend our time with and who not to and how much time we devote to a certain individual. Socialization is thus an expression of economic calculation and therefore being liked is connected with higher social status than being totally asocial.

Hobbes' model of society doesn't recognize natural affiliation toward others, instead its core lies in purely egoistic behavior. Why (how?) did the society come about then?

Hobbes thinks it came about because (while disregarding the inability of humans to take care of themselves since birth) even the strongest and most skilled individual eventually comes to the conclusion that living with others is preferable to living alone. In a thought experiment which is the cornerstone of social contract concepts he refers to natural status as *bellum omnium contra omnes* (the war of all against all), that is a status in which Plautus'*homo homini lupus* applies (man is a wolf to his fellow man). This status can be characterized by total freedom and equality of all participants. It is in fact a state of a non–cooperative game of individuals — a game for life. Each player strives for his own survival and for maximization of payoffs stemming from implemented strategies. And it becomes quite apparent that in an attempt to maximize one's own profits (or to increase the likelihood of gaining them) it is preferable for players to abandon the non–cooperative strategy and use the cooperative one instead. Even the strongest and the healthiest individual realizes, that one day he may become unfit and old and that he must sleep and can't hunt and stay in his cave at the same time. The war of all against all is disadvantageous and unsustainable in the long run and thus players abandon some strategies leading to short term profits providing other players also abandon such strategies (*reciprocity*). In this way the player maximizes his payoffs in the long run. According to Hobbes, the social contract and cooperative behavior are demands of common sense and thus should become the law for rational beings (Démuth, 2011, 53 — 59) — the law of not using strategies prohibiting future cooperation (Kant, 1996).

10.2 Utilitarianism

The problem of the social game, however, lies in the fact that it is a game with an indefinite number of players and an equally indistinct number of ties between them. Furthermore, each player can have, and often has, specific targets and preferences. This creates a problem of *social choice* and individual preferences. Classic *utilitarianism* presented by David Hume, Adam Smith and most notably by Jeremy Bentham introduces society as a non–monolithic group of beings (as opposed to Hobbes' Leviathan) possessing different individual preferences, which can often lead to *Condorcet's paradox of choice* (A prefers B, B prefers C, C prefers A) generalized into a form of *Arrow's impossibility theorem*. Therefore individuals in society create ad hoc coalitions leading to fulfillment of their individual targets and preferences.

One of the key issues of social games is the question of specificity and measurability of winnings. Classic utilitarianism supposes that we strive for happiness, well–being and common good and that a good game is a game which ensures the greatest possible happiness for the greatest number of individuals. Unfortunately, none of the philosophers has yet defined common good in a way, in which the definition would not possess the traits of tautology and at the same time could claim general validity (Moore, 1988). Smith's and Bentham's tradition supposes that by pursuing one's own objectives we generate common good (*summum bonum*). Is there, however, in a social–economic game of diversely oriented individuals any common good?

10.3 Utility and common good

Bentham, von Neumann and Morgenstern defined it as utility. Lionel Robins, however, objected to the cardinal utility theory and said, that well–being, happiness and utility are mental states of individuals, which are not comparable with one another and thus there can't be any common good. John Harsanyi (winner of the Nobel Prize in economics in 1994 for GT and equilibriums in non–cooperative games) agrees with Robins that full comparability and equality of mental states such as utility is never possible, but believes, however, that human beings are able to make some interpersonal comparisons of utility because they share some common backgrounds, cultural experiences, psychological and ethical values etc. Common good is then a result of common activity leading to a goal, which would have been preferred by the most rational being acting on behalf of other players (philosopher, leader). At the same time (from an individual's point of view) it is a consequence to which this individual would have arrived, had he been *Arrow's dictator* (Binmore, 2011, 64). This is the core of Harsanyi's doctrine, and that is: two rational beings in an identical situation arrive at identical decisions. (As opposed to Nozick's *paradox of twins,* where identical decisions and an agreement are impossible — Nozick, 1969).

Amartya Kuman Sen — winner of the Nobel Prize in economics (1998) for his theory of welfare and poverty mechanism — believes in the global ability of utility comparison based on comparing the whole context, in which utility develops. However, this comparison must make provisions for the starting point as well as all possible utilities that are being presented to the individual. This results in the possibility of creating coalitions even among totally diverse social groups or individuals with different interests (because although they pursue different possible utilities based on different reasons, for all members of a coalition these utilities are desired) and these coalitions can be created only at the expense of other groups' interests (*Pareto's liberalism paradox*). Where does the pro–social behavior and the idea of common utility come from, then?

10.4 Emphatic preferences

Kenneth George "Ken" Binmore is convinced that the struggle to increase the chances of one's own profit is the very thing that leads individuals toward something, which he calls *emphatic preferences*.

> We empathize with others in order to understand their future decisions and based on this create our coalitions. Empathizing means to imagine oneself in someone else's place, in his/her social, psychological and value context.

This is precisely the thing that autistic individuals lack. They don't possess sufficient levels of social feelings. As stated in Dennett's style: the mind establishes a concept of intentional strategies and an image of intentionality of others in order to improve interactions in the world. Sellars's myth about Johnson and Damasio's concept of consciousness, in which emotionality precedes rational processes are paradoxes of this concept. Utilitarian modeling of another person's mind is the keystone of a rational–utilitarian concept of society. Based on empathy we are able to predict behavior of another individual and empathy creates social feelings, which can manifest themselves in the form of affinity (as a harmony between individuals) or aversion or hostility or a whole other scope of social and emotional states and phenomena.

The basic idea of GT as a social contract is the understanding of society as a social game. The basic task of social philosophers, politicians and lawyers is to create such rules (laws and mechanisms), which enable optimization of payoffs stemming from participation

in the social game and minimization of potential losses brought on by the use of illegal strategies (punishments) or by behavior rendering future co–existence impossible.

10.5 Recommended literature

BINMORE, K.: *Game Theory and the Social Contract I. Playing Fair*. Cambridge: MIT Press 1994, 242 — 253.

BINMORE, K.: *Game Theory and the Social Contract II*. Cambridge: MIT Press 1998, 147 — 167.

11. GT and morality

Key words: *selfishness, altruism, virtue, reciprocity, kinship*

There are frequent objections to the understanding of society as a product of rational choice of individuals. These are arguments stating that humans in society don't in fact make rational decisions (more to the contrary — limited rationality), but they are subject to their surrounding environment and macrostructures. (Marsh — Stoker). They abide not only by their own interests, but also take into account the interests of others in a sense that rationality doesn't determine emotions, but on the contrary is a product of emotions (Damasio), but mainly it takes into consideration the fact that besides pure egoism humans also display pro–social behavior based on other principles than just their own benefit (Sedláček).

The last argument about existence of social affiliations stemming from sources other than egoism is as old as philosophy itself. However, it was specifically debated in an English Moral Sense School, where the 3rd Lord of Shaftesbury, Francis Hutcheson, and other Enlightenment thinkers pointed out that the lack of a self–preservation instinct poses a threat to the species survival and at the same time warned that an excessive affinity can lead to the downfall of an individual. They considered both of these instincts to be natural to humans, but only the unifications of both principles and their mutual balance seemed truly moral. Similarly Tomáš Sedláček (2012, 51) revisits the Aquinas teachings about the need of utilizing evil (egoism) in service of the common good (love) and

it is love that is often considered to be the unmistakable proof of deficiency of a rationally economic theory of morality. The more so that even Adam Smith himself, the author of the invisible hand of the market, didn't suppose that society can be built purely by pursuing one's own interests. The core of morality, according to him, lies in (as stated in his *Theory of Moral Sentiments*) empathy, listening to others and affection, that is in an ability to sympathize with others. The issue of synchronizing both works *The Wealth of Nations* and *The Theory of Moral Sentiments* opens up a question referred to as "*das Adam Smith Problem*". Is love really so different from self–love and is it necessary to base ethics on two contrarily functioning principles?

11.1 The selfish gene

Perhaps the most thorough, but also the most controversial, application of GT (in the area of explanation and building of morality) can be found in the works of Richard Dawkins and his followers.

Dawkins claims, that the real participants of games with Nature are genes. All virtual individual players are just products of genes. What genes are really after is simple survival and thus maximization of future payoffs' probability. That is why selfish genes come up with various strategies of self–preservation. These can be divided into two basic categories — 1. stability secured by durability, or by large numbers, 2. durability secured by replication and flexibility. While the first strategy is used mainly in simple forms of life (bacteria, viruses etc.), the second strategy led genes to creation of more complicated conglomerates of entities and to their mutual cooperation.

The essential feature of neo–Darwinistic interpretation is the fact, that existence of multicellular organisms is based on a selfish strategy of genes. For their preservation genes form complex biological automats — the bodies. Consequently, they use these bodies to look after themselves and for self–replication, which renders them almost immortal. Part of their strategy is an adaptation to their surrounding environment as well as the opposite mechanism — shaping and formation of the environment — the culture. And not only the tangible one, but also the intangible one — laws and rules. Dawkins thus distinguishes between genetic (gene) and non–genetic (meme) replicators and the law or moral rules are subject of *memetics*. Similarly to Nature, where only the more successful life forms survive, in culture only those rules, which prove to be effective and help individuals to succeed, remain.

Neo–Darwinistic and pragmatic approaches suggest a link between morality and biology, but moral categories can't be fully reduced to it. Ken Binmore, therefore, compares morality and moral feelings to Chomsky's generative grammar or Pinker's language instinct. This instinct is differently realized in different linguistically stimulating environments. Similarly, deeper — perhaps innate — *moral and legal structures and instincts* are differently determined by social evolution (Binmore, 1998, 181).

Matt Ridley and other proponents of neo–Darwinism (William Hamilton, George Williams) are convinced that morality stems from a self–preservation instinct and from rationalization while utilizing egoism and an effort to survive (Ridley, 2000, 30, Ridley, 1999). With the use of certain instruments of GT Ridley steers the selfish gene proposition to a conclusion, that

> *altruism and pro–social behavior* are just well *disguised selfishness and a rational strategy* enabling individuals to maximize the probability of highest possible profits in long term iterated games.

Cooperative behavior and sacrificing part of one–off profit (for the benefit of long term payoffs) are the most effective mechanisms in the long run, therefore helping others and altruism are

advantageous strategies of behavior. It can be successfully observed in family and parental relationships.

In Dawkins' and Ridley's view giving birth to children is a way, in which genes move in time. In this sense taking care of a child is in fact an investment into one's own future — into a new body as a means of survival (of genes). Parents sacrifice part of their own means and energy for raising children not only as a consequence of a loan (Tit for tat — children later helping parents), but mainly to enable the survival of one's own genes. That is why affection and altruism are most evident among parents and their offspring.

11.2 Utility transfer rules

An individual (who himself is a product of genes) has understood, that the best strategy for survival of his genes is a creation of physical and formal partnerships and coalitions with other individuals, which is manifested in couples, marriages, tribes and clans. These enable not only the physical survival of genes, but also creation of favorable conditions for smoother survival in games/battles with other individuals from other coalitions. Binmore identifies *(tribal and group) closeness* and *reciprocity* (direct as well as indirect — Binmore, 1998, 183 — 187) as the main rules of creating such coalitions. Tribal closeness is responsible for affiliation based on kinship (in this context J. B. S. Haldane states, that he would die only for his "two brothers or eight cousins" — Binmore, 1998, 188), group closeness causes affiliation based on mutual dependency and reciprocity in rational advantage. Emotionality (world No. 2) and moral norms as part of Popper's world No. 3 are thus in fact just extensions of an unconscious and selfish strategy of genes pursuing their own survival based on *family extensions* and tribal kinship and *intergenerational and inter–social transfers*. Genes and whole individuals (created by genes) thus create and accept memes (laws and moral rules), which enable them to organize themselves in families and consequently (by transferring rules among families) also in society, which renders their existence in the world easier.

11.3 Self–love as the basis of altruism

The statement that self–love is the basis of any form of social love can be found in many moral codices. A good example is the famous biblical: "*Love your neighbor as yourself!*" which also presumes the primacy of self–love. GT and social neo–Darwinism are trying to emphasize that morality and the law can be thought of as products of finding the most successful strategies, games and rules of survival. In this sense the history of ethical and legal systems can be perceived as a search for optimal rules leading to the highest possible profits for the largest possible number of attending players. While in the past the harsh rules of survival of the most powerful were preferred, as time went by we could see a shift toward enforcement of rules taking into consideration cooperation and survival of a larger number of players. After all the results of Axelrod's tournament correspond with this tendency. While in the first tournament Hammurabi's or the Old Testament style of "eye for an eye" algorithm succeeded, in the second tournament more forgiving and cooperative variants resembling the message of the New Testament proved more successful.

11.4 Recommended literature

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12. Evolutionary GT

Key words: *evolutionarily stable strategy, mutant strategy, monomorphism, replicator dynamics, the speed of convergence*

An important message of different thoughts about GT is the conviction about their evolutionary character. John Maynard Smith and George R. Price published in 1973 an article *Logic of Animal Conflict*, (Maynard Smith, Price, 1973, 15 — 18) in which they proved that in interspecies conflicts individuals conduct the Darwinian fight for survival, whereas in conflicts within the same species (such as fight for a mate) individuals more often opt for a game of duel rather than exposing themselves or the opponent to a real threat. This idea motivated Maynard Smith (while utilizing instruments of GT) to formulate a separate theory named *evolutionarily stable strategy* (EES), which became the basis of his work (Maynard Smith, 1982) awarded the Crafoord Prize. This theory introduces both basic moments of evolutionary development: *survival of the more successful* and *dynamic refinement of the system.*

12.1 Evolutionarily stable strategy

> Maynard Smith's evolutionarily stable strategy is an optimal — dominant strategy of a certain species in a particular environment. Deviations from EES are designated as mutant

strategies and it applies that the total payoffs from ESS always exceed payoffs from mutant strategies.

Mathematical and probabilistic interpretation of behavior of certain species became the spotlight of much ethological research. GT thus provides a tool for explanation of sexual behavior of many mammals (Ridley, 2001), reproductive behavior of fish or birds, fighting strategies of spiders (Riechert, Hammerstein, 1983, 377 — 409, Riechert, 1984, 1 — 15), understanding of ecological problems, etc. In evolutionary GT, payoffs are characterized as biological or reproductive abilities.

Darwin's theory of evolution supposes that species which are better adjusted to the conditions of a particular environment will be more successful than those whose biological predisposition prevents them from succeeding in the fight for survival. As a result, systems with optimal decision and integration strategies last while others are limited or die out. This applies to both biological and cultural systems such as theories, laws or rules. Evolutionary understanding of GT can thus be used to predict the success rate of certain cultural, legal or scientific theories or approaches.

The essential feature of ESS is its stability or stasis. Neo–Darwinists describe the behavior of individuals of the same species (monomorphic society) in an environment with relatively invariable conditions while supposing limited rationality of individuals, minimal or zero degree of awareness of reasons for certain strategies as well as hereditary succession of these strategies. The stability of conditions and of employed strategies eventually leads to establishment of equilibrium in terms of Nash equilibrium.

12.2 Dynamic analysis of ESS

The weak point of ESS is the difficulty of explaining changes and etiology of biological or social occurrences (McKenzie, 2009). Dynamic analysis of ESS supposes a change in environmental conditions, the influence of selective pressure in changing conditions and an existence of mutations and mutant strategies. In biological evolution the most frequent changes are brought by genetic mutations and selective pressure of the environment (Darwin's version of evolution). The frequency and faultiness of replications — multitude of mutations — plays an important role in this process.

For social and cultural phenomena it applies that mutations can be brought by the variation of the environment, by chance (trial/error) as well as deliberately — by pondering and imitation (Lamarck's version). The factor expressing the probability of behavior change of a player with a lower payoff value as that of the payoff value of his opponent (based on comparing an opponent's payoffs or on external conditions) is called *replicator dynamics*. The replicator dynamics expresses the degree of instability of ESS and the probability of its change in a given system. While genetic replications are subject to a relatively small degree of change in development dynamics (depending on the number of replications) in a cultural area is change determined mainly by the quality of interactions, imitation and learning.

In general we can say that in both biological as well as socio–cultural evolution, Nature (as a player of GT) is the main dynamic factor, therefore the presence of awareness and reflection of used strategies is highly questionable.

Some philosophers believe in eternity and the innateness of logical, esthetic, and ethical norms and they perceive them as natural laws, but most see them as being influenced by education and experience. In the spirit of evolutionary epistemology "apriorism" and conviction about innate instincts can also be explained by evolution and GT. In such an approach logical, esthetic and ethical norms are just generically acquired patterns of behavior, which enabled (or disallowed) certain communities to achieve payoff functions (social ability and effective survival at the expense of other communities). And that is despite the fact that an individual might not be consciously aware of such function.

Nature can be thought of as rational, searching for optimal, stable equilibrium between its particular entities. The speed in which this equilibrium can be achieved is called *convergence speed*. If we accept the validity of Dawkins's theory of a selfish gene (biological automatic replicator), which supposes accidental and experimental testing of various biological or cultural strategies (meme), GT can then be successfully extended also into non–living Nature and it can be applied among other (non–biological) automats. With respect to the limited rationality of humans or to the fact that we are not just rational systems, we can expect more effective regulation of non–human artificial systems as well as their mutual influence and evolution.

12.3 Applicability of ESS

As an example of using GT in this area we can mention management of transport systems, network administration and many other situations, in which different users can opt for different strategies of behavior. Based on the fact that the success rate and selective pressure are given by a human user's interests, we can expect that mechanisms similar to mechanisms of biological evolution and of free market theory would be effective here as well. If we leave traffic without traffic rules or without sanctions for breaking the rules, most players will opt for strategies leading to the most successful dealing with traffic. What will certainly follow are unwritten rules and strategies and gradual elimination of numerous conflicts. On the other hand, it is quite probable that the number of those taking advantage of such a system at the expense of others (misuse of shortcuts, crowded roads etc.) will rise. Hence, when managing

transport systems and building competitive transit arteries is impossible, systems regulated by an intelligent administrator seem (in situations of uneven transport) more successful than those leaving the regulation of the system to the invisible hand of the market and that's mainly because they eliminate time needed for agreement about modification of used rules in changing situations (they increase the convergence speed) plus they eliminate "clever" strategies leading to optimization of private payoffs at the expense of public costs. Similarly the use of common networks naturally supports establishment of asymmetric relations between the user and the administrator, who use different strategies dependent of their different roles. Equality of conditions thus unwittingly produces diversity of roles.

Evolutionary GT presumes development and deepening of rationality of all (even artificial) participating systems. Based on the fact that players rarely participate in games with complete information, one of the most important recommendations is to opt for decisions and steps which enable players to make provisions for uncertain and improbable conditions and occurrences or (in the spirit of Popper's falsification theory) they afford opportunity for correction of strategies players had already chosen. In the field of economy we can talk about diversification of sources and risk elimination, in other fields about adequate redundancy and in law, philosophy or politics about making decisions whose potential fallaciousness will not lead to disastrous or irreversible consequences. With respect to limitations in knowledge we can't preclude the principal faultiness of our strategies and that's exactly the reason why GT doesn't provide tools for normative judgments (how we should decide), but rather tools for understanding of what can happen (and with what probability) if we decide in a certain way.

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